updated 15th March 2020

MATH 101:201 Integral Calculus with Applications to Physical Sciences and Engineering

lecture summaries

spring term 2019/2020

MWF 9:00-9:50 (Chemistry B250)

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Integration

M 06/01 | 1 | Introduction to integration (lecture given by Anthony Wachs)

- Introduction: differentiation versus integration (1.1)
- Review: summation notation (1.1.2)

W 08/01 | 2 | Definite integrals (lecture given by Anthony Wachs)

- Definition: Riemann sums and the definite integral (0.4)
- Geometric interpretation of integrals as signed area (1.1.3)

F 10/01 | 3 | Basic computations

- Review: definite integrals as limits of Riemann sums
- Computing integrals using areas of known shapes (1.1.4)
- Basic properties of the definite integral (1.2)

M 13/01 | 4 | The fundamental theorem of calculus I

- Computing integrals of even and odd functions (1.1.4)
- A physical interpretation of integrals (1.1.5)
- The fundamental theorem of calculus (FTC) version 1 (theorem 1.3.1, part 2)
- Examples: reading differentiation tables "upside-down" (examples 1.3.11ff)

W 15/01 | ★ | Class cancelled due to "extreme weather" (a bit of snow)

F 17/01 | 5 | The fundamental theorem of calculus II

- More examples: reading differentiation tables "upside-down" (examples 1.3.11ff)
- The fundamental theorem of calculus (FTC) version 2 (theorem 1.3.1, part 1)
- The FTC and the chain rule (Examples 1.3.2–1.3.6)
- More examples about how to use the FTC: in particular $\int \frac{1}{x}\,\mathrm{d}x = \ln|x|$ (examples 1.3.13)
- Examples about how *not* to use the FTC: in particular, checking domains and continuity!

M 20/01 | 6 | Substitution rule for indefinite integral

- Revision: Linearity of integration and differentiation (1.4)
- Substitution rule from chain rule for the indefinite integral; many examples (theorem 1.4.2ff)

W 22/01 | 7 | Substitution rule for definite integral and area between curves

- Substitution rule for definite integral; examples (theorem 1.4.6)
- Desmos presentation: area between curves (see announcement on Canvas)
- Signed area versus total area between two curves; examples (section 1.5)

F 24/01 | 8 | Area between curves

- Example: total area between two curves that intersect multiple times (see announcement on Canvas)
- Example: integration in y-direction is sometimes easier (section 1.5)

M 27/01 | 9 | Volumes

- Volume = integrated area (1.6)
- Examples: volume of cone, sphere, pyramid, torus (1.6)
- Exercise: cones of arbitrary shapes

W 29/01 | 10 | Volumes (continued) and averages

- Computation for torus (continued)
- Volume of the perpendicular intersection of two cylinders of unit radius
- Volume of a solid obtained by rotation about another line than simply $x = 0$ or $y = 0$
- Definition and example: the average of a function (2.2)

F 31/01 | 11 | Work

- Definitions and basic examples, in particular: Newton's second law and Hooke's law (2.1.2)
- Example: pumping water out of a conical tank (2.1.4)

M 03/02 | 12 | Integration by parts (IBP)

- example on work (continued) (2.1.4)
- Theorem and many examples (1.7)
- IBP Trick 1: multiply the integrand by 1 (Examples 1.7.8 and 1.7.9)

W 05/02 | **13** | **IBP (continued) and trigonometric integrals with powers of sin and cos**

- IBP Trick 2: Apply IBP twice and then solve for the integral (Examples 1.7.10 and 1.7.11)
- General algorithm for solving $\int \sin^a(x)\cos^b(x)\,\mathrm{d}x$ and many examples (1.8, 1.8.1)

F 07/02 | **14** | **Trigonometric integrals with powers of sec and tan**

- General algorithm for solving $\int \sec^a(x)\tan^b(x)\,\mathrm{d}x$ and many examples (1.8.2)

M 10/02 | **15** | **Trigonometric substitutions I**

- The three prototypical examples: $\int\sqrt{a^2-x^2}\mathrm{d}x$, $\int\sqrt{x^2-a^2}\mathrm{d}x$ ad $\int\sqrt{x^2+a^2}\mathrm{d}x$ (1.9)
- Undoing the substitution after successful integration (1.9.3, 1.9.5, 1.9.6)
- Importance of pausing for a moment before ploughing on (example 1.9.4)

W 12/02 | **16** | **Trigonometric substitutions II**

- Completing the square before trigonometric substitutions (example 1.9.7)
- Integrating simple rational functions without partial fractions

F 14/02 | ★ | **Midterm exam**

17–21/02 | ★ | **Midterm break**

M 24/02 | **17** | **Partial fractions I**

- Polynomial division (1.10)
- Factorizing quadratic polynomials by guessing the roots (A.16)

W 26/02 | **18** | **Partial fractions II**

- Partial fraction decomposition in examples (1.10)

Applications of integration

F 28/02 | **19** | **Numerical integration (This lecture is non-examinable.)**

- Approximations via Taylor polynomials
- Approximations via midpoint rule, trapezoid rule and Simpson's rule (1.11.1–1.11.3)
- Comparison of error terms for midpoint rule, trapezoid rule and Simpson's rule (1.11.4)

M 02/03 | **20** | **Improper integrals I**

- Improper integrals with unbounded limits of integration (1.12.1-2)
- Improper integrals with unbounded functions (1.12.1-2)

W 04/03 | **21** | **Improper integrals II**

- More examples: improper integrals with unbounded functions (1.12.2)
- Convergence tests for improper integrals (1.12.3)

F 06/03 | **22** | **Centres of mass**

- Centres of mass: 1-dimensional case (example: centre of mass of a cone) (2.3.1)
- Centres of mass: 2-dimensional case (example: inverted parabola) (2.3.1)

M 09/03 | **23** | **Differential equations**

- A primer on differential equations (2.4)
- Topical review: exponential growth (see 3Blue1Brown)
- Separable differential equations (2.4.1)
- Exam revision strategies (see the revision checklist)

Sequences and Series

W 11/03 | **24** | **Sequences and series**

- Definition and examples: sequences, convergence, divergence (3.1)
- General principles and arithmetics for limits of sequences (3.1)
- New from old: series and their limits (3.2)
- Geometric series (3.2)

F 13/03 | **25** | **Lots of series**

- Arithmetics of series, telescoping series (3.2+3.3.9/warning 3.3.3)
- Divergence test: $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ (harmonic series) versus $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (Basel problem) (3.3)

— COVID-19 —

Starting Monday 16/03, all lectures are moving online for the remainder of the term. Please see Anthony Wachs's email for details.